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The propagation of cracks in composites consisting of ductile wires in a brittle matrix

J. BOWLING*, G. W. GROVES
Department of Metallurgy and Science of Materials, Oxford University, UK

Two types of fracture toughness specimens, double cantilever beam and single-edge notch, were constructed from nickel wires in an epoxy resin matrix. The critical stress intensity factor to cause propagation of an unbridged matrix crack arrested at a wire was found to be greater than that for plain resin. Subsequent crack propagation could be described by taking account of the stress intensity factor due to known forces in the crack-bridging wires which subtracted from that due to the applied forces. The debonding and pull-out behaviour observed previously in single-wire specimens was confirmed in the multi-wire fracture toughness specimens.

1. Introduction

In a previous paper [1] a detailed study was made of the debonding and pull-out of individual nickel wires from a resin or cement matrix showing that debonding of the wire occurred at a stress slightly in excess of the yield stress, with a plastic strain dependent on the roughness of the surface of the wire. Debonding thus occurred over a substantial crack opening, at a very predictable stress. In this paper it is shown the same effects arise in multi-wire composites and that the known forces carried by the debonding wires or wires pulling out which bridge a propagating crack in the composite can be used to predict the applied forces needed to fracture a pre-cracked composite.

2. Experimental details

Fracture-toughness specimens of two forms were prepared; double cantilever beam (DCB) and single-edge notched (SEN). The form of the DCB specimens is shown in Fig. 1. The dimensions were $b = 6$ mm, $w = 2$ mm, $d = 25$ mm. SEN specimens were 5.8 mm thick, 90 mm wide and 300 mm long. Nickel wires of the type used in a previous study of a single wire debonding and pulling-out [1] were incorporated to various volume fractions in an epoxy resin matrix (Araldite type MY753 with hardener HY951). The wires were placed at the centre of the thickness of the DCB specimen and alternately $\frac{1}{4}$ and $\frac{3}{4}$ the way through the thickness of the SEN specimen; and were spaced regularly to give the required volume fraction.

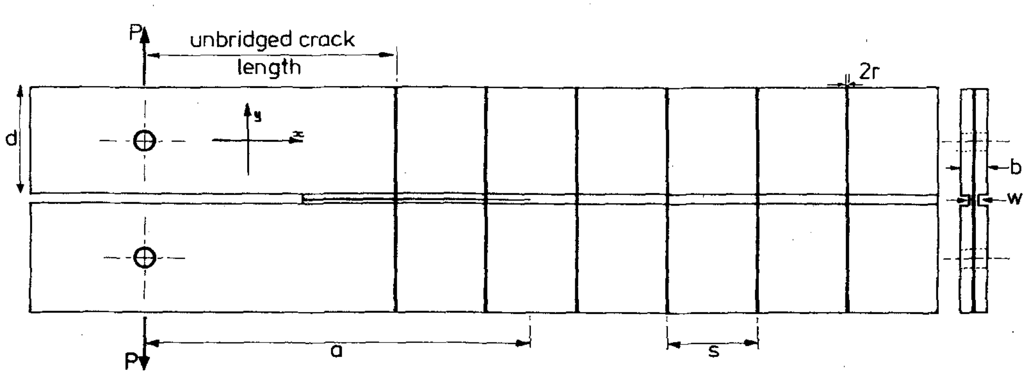

Figure 1 Specimen geometry for double cantilever beam specimen.

*Present address: I.M.I. Ltd, Kynoch, Birmingham, UK.

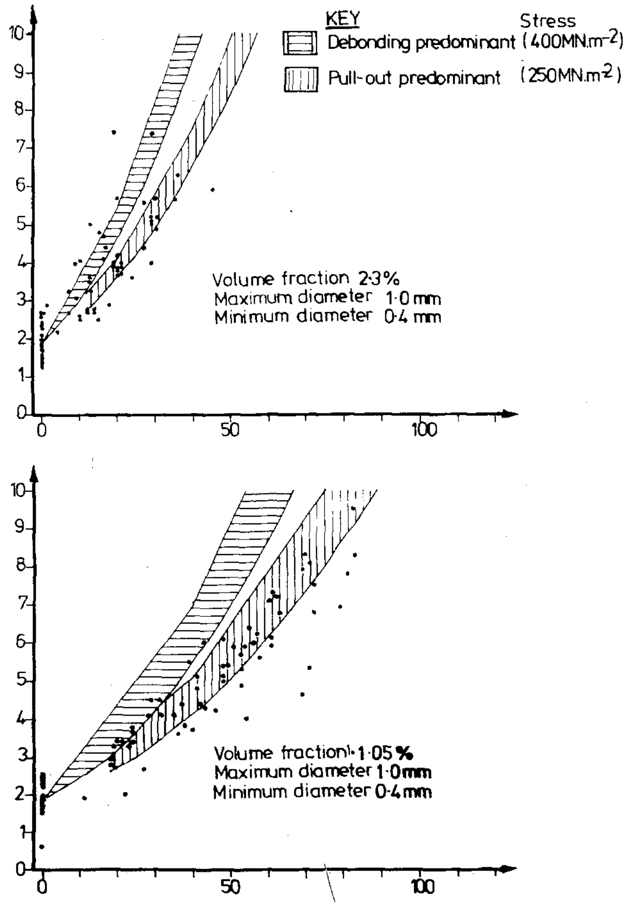

Figure 2 Measured critical stress intensity factors for two volume fractions of nickel wire (yield stress $400\,\mathrm{MN\,m^{-2}}$) in epoxy resin compared to theoretical lines.

3. Results

3.1. DCB specimens

Four volume fractions of wire were employed, the largest being 0.023. Specimens of resin only were also tested to determine the critical stress intensity factor for the matrix. This was independent of crack length and was $0.95\,\mathrm{MN\,m^{-3/2}}$.

The initial crack, introduced with a razor blade, halted at the first wire in the composite. The critical stress intensity factor for the propagation of this crack did not vary significantly with wire diameter in the range 0.4 to 1.0 mm and was $1.86\,\mathrm{MN\,m^{-3/2}}$. Since there was only one wire at the crack front for all volume fractions, different volume fractions being obtained by varying the wire spacings along the specimen, there was no effect of wire volume fraction on this parameter.

As the crack propagated in the DCB specimen and became bridged by wires, the measured critical stress intensity factor, for further propagation, increased.

This increase is clearly due to the presence of wires bridging the crack and exerting closure forces which reduce the actual stress intensity factor pertaining to the matrix crack tip. Thus a stress intensity factor K_w must be subtracted from that due to the applied force, K_a and the net stress intensity factor $K_\mathrm{a} - K_\mathrm{w}$ must reach a critical value K_Ic for crack propagation.

Thus

$$K_\mathrm{a} - K_\mathrm{w} = K_\mathrm{Ic}. \tag{1}$$

The value of K_Ic is taken to be that for propagating an unbridged crack past a wire, since subsequent arrests of the crack generally occur at a wire. For the DCB specimen [2]

$$K_\mathrm{a} = \frac{Pa}{w^{1/2}b^{1/2}d^{3/2}}(3.467 + 2.315\,d/a), \tag{2}$$

where the symbols are defined in Fig. 1. The force P is measured at the moment of crack propagation. The contribution of each bridging wire to K_w is calculated by substituting for P in Equation 2 the tensile force in the wire and for a the distance from the wire to the crack tip. When the crack has passed several wires in the DCB specimen, the relatively large crack openings for this specimen cause the wires furthest from the crack tip to pull out at the instant of crack propagation. The value of K_W was therefore calculated for the limiting cases of (a) the wires carrying the debonding load, and (b) the wires carrying the pull-out load. A comparison of the experimental results with the predictions of Equation 1 is shown in Fig. 2, for the two highest volume fractions of wire. The bands of predicted values correspond to the range of wire diameters, the larger wire diameters giving slightly larger values of K_W by concentrating the force in larger discrete units. Unfortunately, the scatter in the experimental results is too large to allow a very critical comparison with theory but the general trend of increasing K_a with increasing bridged crack length is adequately explained. The pull-out force appears to give rather better agreement than the debonding force, as might be expected, since the wires furthest from the crack tip, which pull out at large crack lengths, make the largest contribution to K_W.

A significant feature of crack propagation in the specimens of highest wire volume fraction was that ultimately the crack deflected out of the groove, causing an arm of the DCB to break off. The deflection originated at a wire but did not generally follow the wire–matrix interface.

3.2. SEN specimens

In these specimens, the crack openings are relatively small compared to those in the DCB specimen and the matrix crack propagates right across the specimen before the debonding of the wires is completed. Four volume fractions of 0.8 mm diameter nickel wire were studied; 0.016, 0.031, 0.048 and 0.064; and wire diameters in the range 0.5 to 1.0 mm were studied at a volume fraction of 0.032. In the case of the smallest volume fraction, the crack propagated catastrophically right across the specimen, but in the other specimens, crack arrests occurred and several values of the stress intensity factor needed for further propagation could be obtained, from the formulae

$$K = \sigma a^{1/2}\,[1.99 - 0.41\,a/b + 18.7\,(a/b)^2 - 38.48\,(a/b)^3 + 53.85\,(a/b)^4] \qquad (3)$$

for $a/b < 0.3$, a being the crack length and b the specimen width and

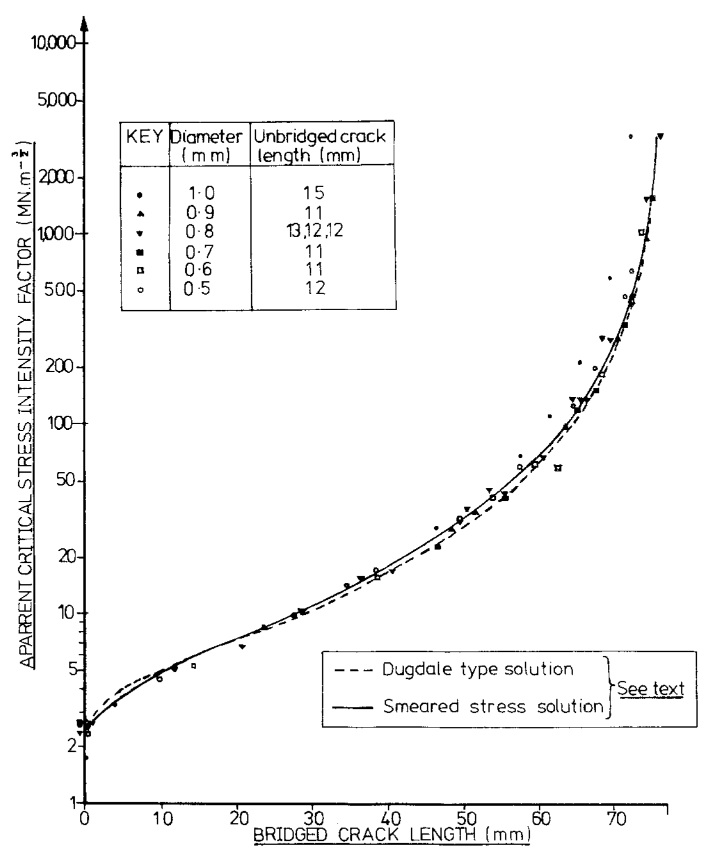

Figure 3 Apparent critical stress intensity factor versus bridged crack length for SEN specimens with 3.2% nickel wire (yield stress 400 MN m^{-2}) in epoxy resin.

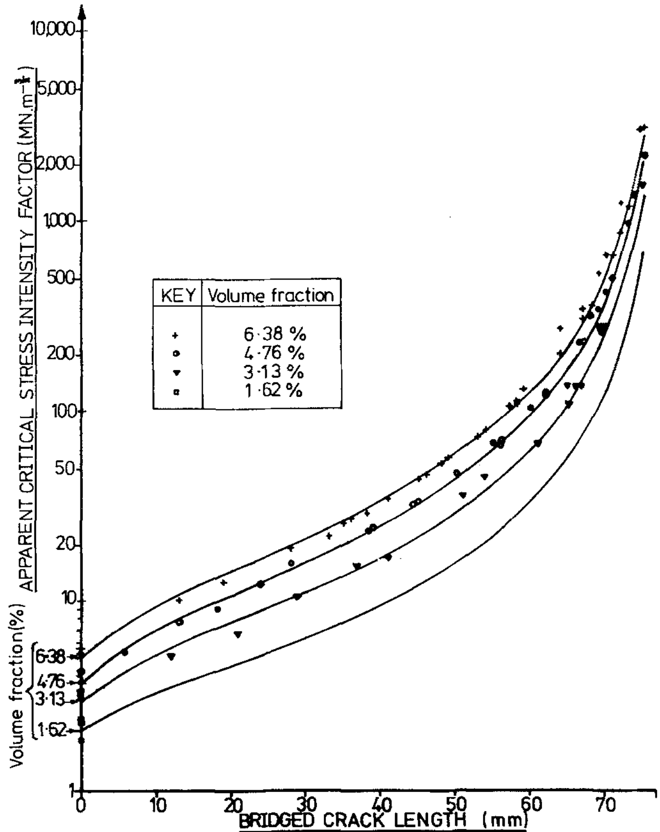

Figure 4 Apparent critical stress intensity factor versus bridged crack length for SEN specimens with four volume fractions of 0.8 mm diameter nickel wire (yield stress 400 MN m^{-2}) in epoxy resin.

$$K = \sigma a^{1/2} \left[\frac{1 + 3(a/b)}{2(a/b)^{1/2}(1 - a/b)^{3/2}}\right] \quad (4)$$

for $a/b > 0.3$ [3]. In Equations 3 and 4 σ is the applied tensile stress. The results are shown in Figs. 3 and 4. The theoretical lines which are seen to closely fit the experimental points are calculated from Equation 1. To make an accurate calculation of the value of K_w, the stress intensity factor due to the wires, account was taken of the fact the initial 10 mm of the crack are, in our case, free of wires, this region being left for the insertion of the initial crack. The force due to the wires crossing the remainder of the crack is then "smeared out" to give an average closure stress; using the debonding stresses evaluated in single-wire experiments [1]. This model is then similar to that of a Dugdale crack with the yielded zone of the Dugdale model represented by the bridged region of the crack in our case. The necessary calculations for evaluating K_w have been performed by Hayes and Williams [4]. However, in our case, the unbridged length of crack is relatively small and it was found that a good approximation to the more exact value of K_w could be made by smearing out the wire forces over the entire crack surface. The smeared out stress, S, is given by

$$S = \frac{(a - c)}{a} V_f \sigma_d, \quad (5)$$

where a is the total crack length, c the unbridged length, V_f the wire volume fraction, and σ_d the debonding stress. S can then be substituted into Equation 3 or 4 to give K_w. Fig. 3 shows that this approximation is very adequate and the lines in Fig. 4 were calculated using Equations 5 and 1. The value of K_{Ic} used in Equation 1 is that for propagation of the matrix crack past the first wires and is substantially in excess of K_{Ic} for the matrix alone, as in the case of the DCB specimen.

Following the complete cracking of the matrix, the wires debonded fully as the crack continued to open and then pulled out at a lower load (Fig. 5). The constant pull-out load is predicted by the model of the debonding process presented previously [1], in which an undeformed plug at the end of the wire is the only part of the wire on which an interfacial shear stress acts. The presence of such a plug was confirmed by direct measure-

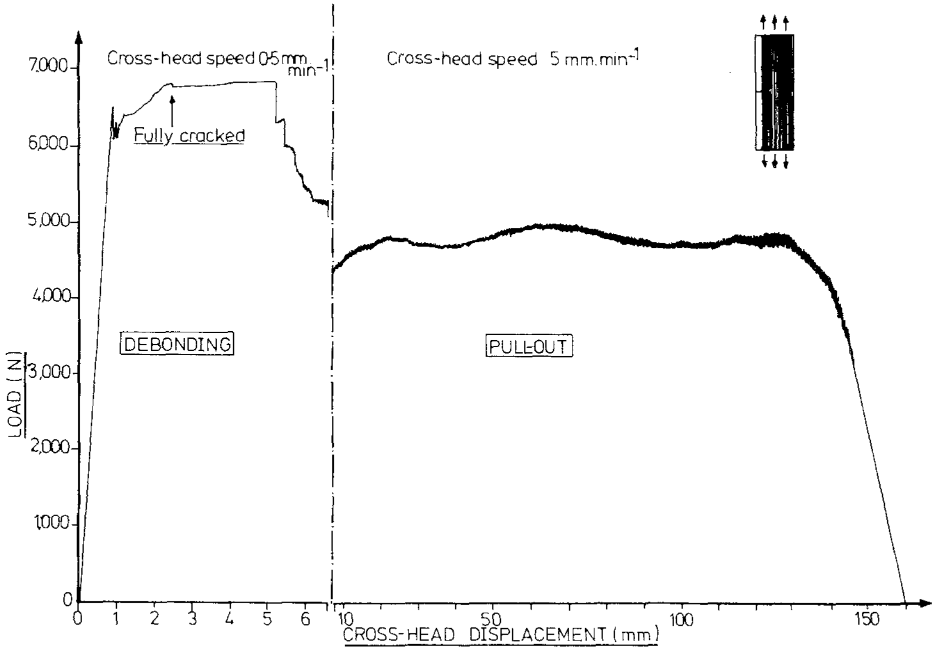

Figure 5 Load–displacement curve for 0.5 mm diameter, 0.032 volume fraction nickel wire (yield stress 400 MN m^{-2}) in epoxy resin.

ment of the diameters of 30 pulled-out wires, which clearly shows the plastic strain suffered during debonding (Fig. 6).

In all cases the total debonding load (Fig. 5) agreed to within 2% with that calculated from the result of single-wire debonding tests [1]. The observed debonding strain of 1.75% for the 0.8 mm diameter wire, obtained from Fig. 6, is consistent with the strains of 1.9 and 1.6% for the debonding of single wires of diameter 0.5 and 1.0 mm, respectively [1]. The pull-out load was consistent with the results of single-wire pull-out tests but the pull-out plateau load was more accurately constant for the multiwire specimen than for single-wire specimens.

From observation of the spacing of photoelastic fringes in the matrix during pull-out it was possible to determine the average interfacial

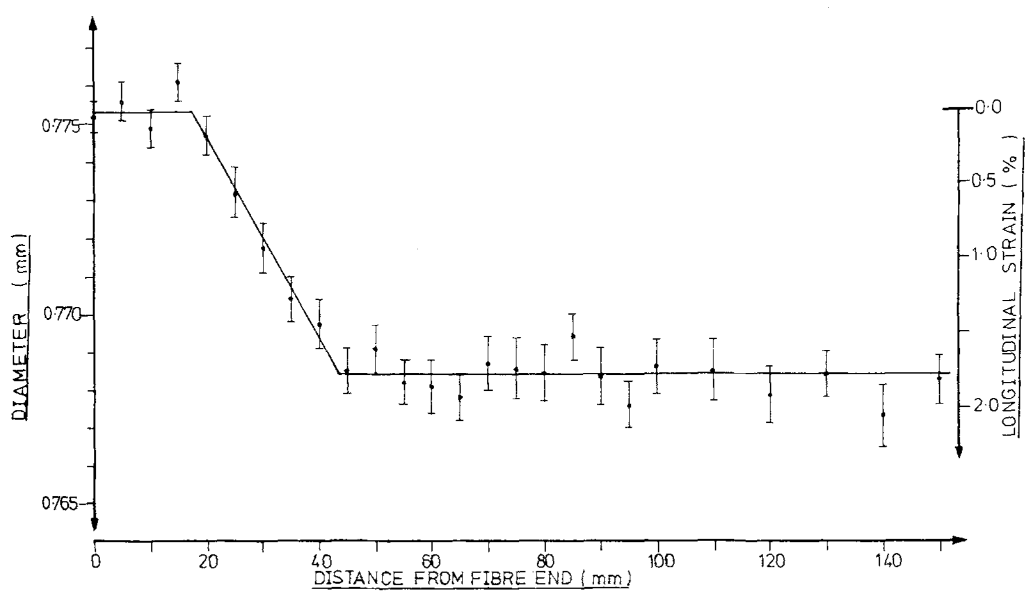

Figure 6 Mean diameter versus distance from fibre end (each point mean of 30 wires, one standard deviation of mean shown) after debonding and pull-out from a 0.032 volume fraction SEN specimen.

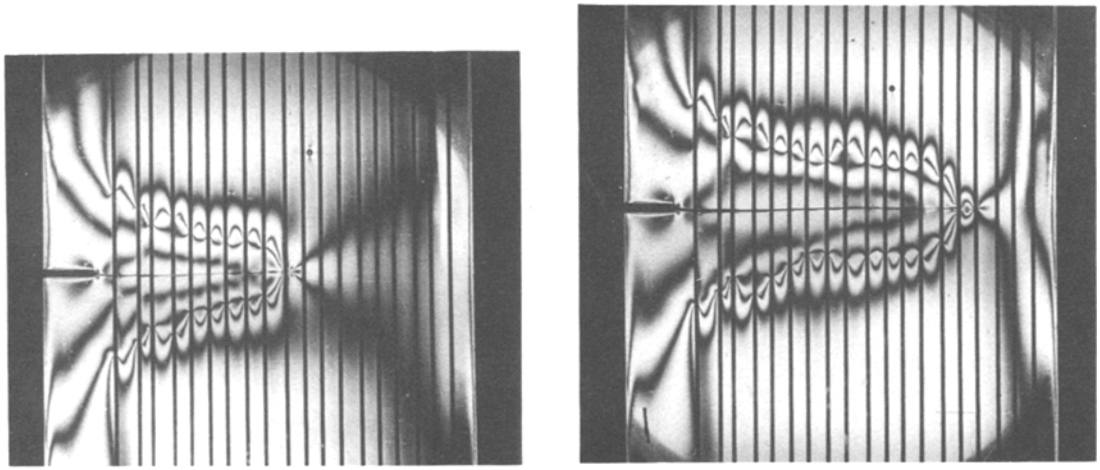

Figure 7 View between crossed polars of the progress of a crack in an SEN specimen. The point on a wire where the photoelastic fringes are most closely spaced marks the position of the debonding front. Volume fraction of wire is 0.032.

shear stress acting on the wire between fringes, by a simple force balance

$$\tau \pi d = \frac{\mathrm{d}\sigma_x}{\mathrm{d}x} wt, \tag{6}$$

where τ is the shear stress, d the wire diameter, w the specimen thickness and t the wire spacing. The matrix stress gradient $\mathrm{d}\sigma_x/\mathrm{d}x$ is given by

$$\mathrm{d}\sigma_x/\mathrm{d}x = \frac{\lambda}{wc\delta} \tag{7}$$

where λ is the wavelength of the light, c the stress optical coefficient of the resin which was measured in a tensile test and δ the fringe spacing. In a specimen containing 0.5 mm diameter wires, the mean shear stress was 2.5 MN m^{-2} over the end 3.7 mm of wire, 2.1 MN m^{-2} over the next 4.0 mm and 1.2 MN m^{-2} over the next 7.6 mm [5]. No further fringes appeared, indicating that the shear stress on the plastically deformed region is very small if not zero.

4. Discussion

In general, the behaviour of the multiwire DCB and SEN fracture toughness specimens can be predicted well from the results of single-wire debonding and pull-out experiments. In particular the change in the applied force needed to propagate the matrix crack with increasing crack length could be predicted well by taking into account the crack closure forces applied by debonding wires or wires undergoing pull-out. The difference in specimen geometry has an important effect here in that the DCB specimen with relatively large crack openings produces wire pull-out whereas the SEN specimen is fully cracked before debonding is complete. To predict an apparent critical stress intensity factor for matrix crack propagation in a composite where wires or fibres are left bridging the matrix crack, each individual specimen's geometry must be considered separately, to take such factors into account. Deflection of the crack from its usual path occurred only in the case of DCB specimens containing the highest volume fraction of wires (0.023), and in this case it was notable that the crack did not generally follow a wire–matrix interface. The deflection of the crack appeared to originate at a wire but thereafter did not follow the interface. Debonding of wires ahead of the crack tip was not observed in our specimens although a wire at the crack tip in an SEN specimen would be debonded over a length of about 1 mm. The position of the debonding front on bridging wires is shown for an SEN specimen in Fig. 7. The debonded length increases rapidly away from the crack tip, confirming that the assumption that each bridged wire has reached its plateau debonding stress is valid.

The feature of crack propagation in multiwire specimens which is most difficult to predict quantitatively is the increase in critical stress intensity factor K_{Ic} needed to propagate an unbridged crack past the first wire, above that needed to propagate a crack in the matrix alone. For the SEN specimen, the increase in elastic modulus due to the wires will produce some increase in K_{Ic} assuming that the same elastic energy release rate G_c is required. Thus, for plane stress

$$K_{Ic} = (EG_c)^{1/2} \tag{8}$$

and

$$\frac{K'_{Ic}}{K_{Ic}} = \left(\frac{E_c}{E_m}\right)^{1/2}, \tag{9}$$

where K'_{Ic} is the critical stress intensity factor for a composite crack (unbridged), K_{Ic} the critical stress intensity factor for the matrix, E_m is the Young's modulus of the matrix and E_c the effective modulus of the composite, as defined by Sih *et al.* [6]. Application of Equation 9 to our specimens showed that the modulus increase could account for only a small part of the increase in K_{Ic}. It must be concluded that there is an increase in the energy requirement for the crack in the composite or a reduction in the elastic energy release arising specifically from the wire at the crack tip. Some increase in energy requirement can be attributed to the fact that as the crack passes the wire it divides onto two different levels, leaving a step where the two sections of crack rejoin at the back of the wire. This gives some increase in surface area, but this is not thought to be large enough to account for the observed increase in K_{Ic}. The greater part of this is believed to be due to the effect of the wire at the crack tip in reducing the elastic energy release, but it is unfortunately difficult to quantify this effect.

5. Conclusions

The fracture of composites of ductile metal wires in a resin matrix has been studied and a model of the debonding and pull-out processes for individual wires has been found to be applicable.

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